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"Parameters affecting the compressive strength and critical strain of masonry"

PARAMETERS AFFECTING THE COMPRESSIVE STRENGTH AND CRITICAL STRAIN OF MASONRY

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To the memory of
Prof. N. Ambrazeys

Summary

The importance of the subject is reiterated, since both design and redesign of masonry structures are mainly based on compressive strength characteristics. The parameters affecting f_{wc} *and* ε_{wcu} *are subsequently enumerated; it is maintained that empirical formulae predicting these basic values should necessarily take into account all the aforementioned parameters. In this paper, for each of these parameters, a short analysis is presented, attempting to offer a possible quantification – very roughly though. Moreover, modifications of* f_{wc} *due to biaxial stress conditions are also discussed; further modifications of these mechanical characteristics under cyclic loading is examined. Finally, a short presentation of the concept of "critical volume of failure" is offered.*

1. SIGNIFICANCE OF THE SUBJECT

Traditional or modern design and re-design methods of masonry structures, are normally based on compressive strength and deformability of masonry: Moduli of elasticity and tensile strength are empirically expressed as functions of f_{wc}, whereas the rotational capacity of masonry in non-linear Analysis is connected to the ultimate strain ε_{wcu} of masonry under uniaxial compression.
Despite their fundamental importance, however, these two characteristics are normally estimated by means of rather doubtful **empirical** expressions – especially in the case of existing older masonry structures. And it may be epistemologically unacceptable to compare action-effects (derived from sophisticated FEM under dynamic loading), with a strength found by means of a lousy empirical formula, pretending for instance to predict the compressive strength of a timber reinforced traditional rubble-stone masonry!
The clear reliability-imbalance of the two sides of the basic safety inequality is apparent. And it should have encouraged intensive research on the resistance-side of the inequality – but this does not seem to be the case, possibly for the following reasons. In fact, such a research:
- should be mathematically overcomplicated;
- it is not amenable to patents, leading to commercial exploitation;
- and, subsequently, it does not attract much funding…

It is however maintained that, because of its basic importance, a concentration of research efforts is needed on the subject; and a broad and longterm european framework should be initiated, aiming at a detailed and fundamental investigation on the structural characteristics of a large variety of masonry types.

This paper is (i) only a reminder of the basic parameters shaping the aforementioned characteristics, and (ii) an attempt for a pseudo-quantification of their influence.

Note: Only **local** strengths are considered; buckling effects are not taken into account in what follows.

2. COMPRESSIVE STRENGTH UNDER STATIC CONDITIONS

In what follows, a description of the composition of masonry will be shortly repeated, a reminder of the parameters affecting the compressive strength of masonry will be given, a theory on compressive strength of rectangular blocks' masonry will be reminded (extended to the case of very irregular blocks), an elementary approximation on the consequences of blocks' bonding will be presented, and finally the role of biaxial stress conditions will be very briefly discussed.

2.1. The composition of masonries

It is of basic importance to recall that the generic term "masonry" is inadequate to describe the enormous variety of possible structural realities – a fact that is not encountered in other building materials, such as timber, steel, concrete or composites. It suffices to enumerate here the large number of the parameters of such a variety:

- Use of mortar, its nature and its quantity.
- Nature, form and bonding of blocks, both in plane or transversally.
- Existence of timber reinforcement.
- Possible three-leaf formation.
- Local variabilities (even within a given wall).

The structural consequences of each of these parameters (and their combination) may be disproportionately large; and this fact is not reflected in the generic term "masonry".
That is why, in the case of existing masonry structures, a **complete** description is needed, backed as much as possible by several in situ measurements (non-destructive tests and imaging methods included). The hope that without such a complete description, "calculations" would be able to predict structural behaviour lies beyond Science.

2.2. Parameters affecting the compressive strength of masonry

It is important to take into account as many as possible of the parameters that affect the strength of masonry under uniaxial compression. Some of these parameters are listed here below, following approximately the order of their importance.

- Bonding of blocks transversally
- Bonding of blocks in-plane
- Strength of the blocks
- Evenness of the seats of blocks
- Normalised thickness of the horizontal layer of the mortar
- Filling of vertical joints
- Strength of the mortar
- Variability of the size of blocks
- Durability conditions.

The state-of-the-art does not allow for a rational calculation of the influence of each of these factors on masonry strength. As a substitute, a very large number of empirical formulae predicting f_{wc} are available, which however take into account a relatively small number of the aforementioned parameters; more frequently, only the strength of the block and the mortar are considered. This may be sufficient for rectangular-blocks' masonries, with perfect transversal bonding. But it is clearly inadequate in a multitude of cases of existing buildings made of masonry with various other typologies.
In situ strength determinations and in-lab measurements on possible replicas (see i.a. T.P. Tassios et al.: "Evaluation of Monuments, Experimental methods and Tests", Int. Symposium SHH07, Antalya, Sept. 2007) may cope with this unhappy situation.

In what follows, however, an attempt is made to offer quasi-quantitative solutions, be it as a mere invitation for more detailed and precise methods to be developed to the same purpose.

2.3. Uniaxial compressive strength of rectangular-blocks' masonry

The main mechanism controlling the integrity of the constituents of masonry in this case, is the generation of horizontal tensile stresses acting on the blocks, because of the larger horizontal displacements of the mortar (due to its Poisson deformation)[1].

With the notations shown in Fig. 1, the transversal strains may be expressed as follows.
Block:

$$\varepsilon_{bx} = \frac{1}{E_b}\left[\sigma_{bx} + v_b\left(\sigma_z - \sigma_{by}\right)\right]$$

$$\varepsilon_{by} = \frac{1}{E_b}\left[\sigma_{by} + v_b\left(\sigma_z - \sigma_{bx}\right)\right]$$

Mortar:

$$\varepsilon_{mx} = \frac{1}{E_m}\left[-\sigma_{mx} + v_m\left(\sigma_z + \sigma_{my}\right)\right]$$

$$\varepsilon_{my} = \frac{1}{E_m}\left[-\sigma_{my} + v_m\left(\sigma_z + \sigma_{mx}\right)\right]$$

where E and *v* denote the elastic constants of blocks (b) and mortar (m), respectively. On the other hand, $\varepsilon_{bx} = \varepsilon_{mx}$ and $\varepsilon_{by} = \varepsilon_{my}$.
For equilibrium, the total tensile force acting on the block should be equal to the total compressive force acting on the mortar, i.e.:
$\sigma_{bx}\, d\, t_b = \sigma_{mx}\, d\, t_m$, thus $\sigma_{bx} = \alpha \,.\, \sigma_{mx}$
$\sigma_{by}\, b\, t_b = \sigma_{my}\, d\, t_m$, thus $\sigma_{by} = \alpha \,.\, \sigma_{my}$
where $\alpha = t_m : t_b < 1$

It results (for $\beta = E_m : E_b < 1$)

$$\sigma_{my} = \sigma_{by} = \left[\frac{\alpha\left(v_m - \beta v_b\right)}{\left(1 - v_m\right) + \alpha\beta\left(1 - v_b\right)}\right] \times \sigma_z = \sigma_t \qquad (1)$$

Because of these horizontal tensile stresses, the vertical compressive strength (f_{bc}) of the block is reduced according to the following approximate expression (see Fig. 2):

$$\frac{\sigma_{zu}}{f_{bc}} + \frac{\sigma_t}{\lambda f_{bc}} = 1 \qquad (2)$$

where
$\lambda = f_{bt} : f_{bc} < 1$

σ_{zu} = the compressive strength of the masonry

Combining Equ. 1 and 2, and putting $\sigma_{zu} = f_{wc}$, we find

[1] The presentation that follows is based on Hendry A.W.: "Structural brickwork", The Macmillan Press, 1981, as modified in T.P. Tassios: "Meccanica delle murature", Liguori, 1988.

$$\frac{f_{wc}}{f_{bc}} = 1 : \left\{ 1 + \frac{\alpha\left(v_m - \beta v_b\right)}{\lambda\left[\left(1 - v_m\right) + \alpha\beta\left(1 - v_b\right)\right]} \right\} \qquad (3)$$

A numerical application of Equ. 3 is shown in Fig. 3 for two qualities of mortar, as follows:
Strong mortar: $\beta = 1{:}3$, $\nu_b = 0{,}15$, $\nu_m = 0{,}25$, $\lambda = 1{:}15$ (where "λ" as in Equ. 3a)
Weak mortar: $\beta = 1{:}10$, $\nu_b = 0{,}15$, $\nu_m = 0{,}35$, $\lambda = 1{:}15$

On the other hand, for a given joint width t_m, the relationship between f_{wc} on the on hand, and f_{bc}, f_{mc} on the other, may be derived from Equ. 3a (simplified version of Equ. 3)

$$\frac{f_{wc}}{f_{bc}} = 1 : \left(1 + \frac{\alpha}{\lambda} \cdot \xi \right) \qquad (3a)$$

where $\xi = \frac{v_m - \beta v_b}{1 - v_m} > 0$, $\alpha = \frac{t_m}{t_b}$, $\beta = \frac{E_m}{E_b}$, $\lambda = \frac{f_{bt}}{f_{bc}}$

Frequently, elastic constants are expressed in terms of respective compressive strengths, e.g. possibly in the following form

$$v = 0{,}5 - k \cdot \sqrt[4]{f_c : f_{c0}}$$

(where "*k*",*fco* and "*n*" are numerical constants)

$$E = n \cdot f_c$$

But, since these "translations" are rather aleatoric, it may be expected that the usual relationships $f_{wc} = \psi\ (f_{bc}, f_{mc})$ could not be precise enough. And this is also the case with simple expressions as the following one, valid for strong and fully bonded brick masonry (Tassios, 1988)

When $f_{bc} > f_{mc}$: $f_{wc} = \left[f_{mc} + 0{,}4\left(f_{bc} - f_{mc}\right)\right] \cdot \left(1 - 0{,}8\sqrt[3]{\alpha}\right)$ (4)

When $f_{mc} > f_{bc}$: $f_{wc} = f_{bc} \cdot \left(1 - 0{,}8\sqrt[3]{\alpha}\right)$

where $\alpha = t_m : t_b$

It is however of a paramount importance to note that neither the field of application, nor the calibration of these expressions are adequately known.

2.4. The case of very irregular stones

As opposed to the previous case, a rubble stone may in extremis be seated on its supporting stone on one point only (Fig. 4), generating transversal splitting stresses σ_{bt}. A pseudo-quantitative solution follows here, only as an indication of the kind of the detailed research needed on the subject.

Assuming an edge support on the contact, splitting stresses could be estimated as

$$\sigma_{bt} \approx 2P : 3h_b \cdot 1$$

On the other hand, flexural tensile stresses "σ_0" may also be superimposed to these splitting stresses. Thus

$$\sigma'_{bt} \approx 2P : 3h_b \cdot 1 + \sigma_0$$

where σ_0 denotes a hardly evaluated stress-increase.
On the other hand, the tensile strength f_{bt} of the stone may in this case be increased thanks to a partial mobilisation of the reaction of mortar filling the vertical joints. Thus

$$eff\ f_{bt} = f_{bt} + k_1 f_{mc}$$

Putting $\sigma'_{bt} = eff f_{bt}$, and $P = l_b \cdot f_{wc} \cdot 1$,
it results

$$f_{wc} = \frac{3}{2}\frac{h_b}{l_b}\left(f_{bt} - \sigma_0 + k_1 f_{mc}\right)$$

Or, for $h_b : l_b \approx 2/3$ and $f_{bt} \approx \frac{2}{3}\sqrt{f_{bc}}$ [MPa],

$$f_{wc} = \frac{2}{3}\sqrt{f_{bc}} - \sigma_0 + k_1 f_{mc} \qquad \text{[MPa]} \qquad (5)$$

This may be a favourable indication regarding the validity of the following **empirical** formula[2], applicable to normally bonded **stone** masonry:

$$f_{wc} = \left\{\frac{2}{3}\sqrt{f_{bc}} - \sigma_0 + k_1 f_{mc}\right\} : \left\{1 + 3,5\left(\frac{V_m}{V_w} - 0,3\right)\right\} \qquad (6)$$

where
f_{bc} = compressive strength (MPa) of the stone
σ_0 = 2,5 MPa, rubble stones
0,5 MPa, semi-carved stones
k_1 = 0,6 for rubble stones
0,2 for semi-carved stones

The denominator is a correction reflecting the detrimental effects of large width mortar joints (described in § 2.3), where:

V_m = mortar volume contained in a masonry volume equal V_w, where $V_m : V_w \nless 0,3$.

2 Tassios P.T., Chronopoulos M.: "Aseismic dimensioning of interventions on low-strength masonry buildings", Middle East Mediterranean Conference "Earthen and low strength masonry buildings", Ankara, 1986.

Once again, it is important to note that the field of application and the calibration of this formula are not well defined.

2.5. Structural consequences of blocks' bonding

Bonding is a basic structural characteristic of masonry. An attempt to quantify a "bonding factor" δ_{bg} is illustrated in Fig. 5. Overlapping lengths "u_i" are summed up, along a through-mortar quasi-vertical section a – a.
Considering approximately "n" layers, such a factor may be defined as follows:

$$\delta_{bg} \equiv \frac{2}{n}\sum\nolimits_1^n \left(u_i : l_{bi}\right) \qquad (7)$$
$$\left(0 \le \delta_{bg} \le 1\right)$$

It is expected that higher δ_{bg}-values ensure better uniformity of internal stresses in masonry and broader possibilities for cracks' arresting.
Until a series of relevant (analytical and experimental) investigations is carried out, an indirect estimator of the significance of bonding will be adopted in what follows: And this will be the vertical compressive stress "σ_{wr}" that initiates quasi-vertical cracking of masonry, (Fig. 6). Since a better bonding corresponds to higher σ_{wr}-values, the ratio of these stresses of two masonries may be considered as equal to the ratios of the ultimate strengths of the same masonries.

$$f'_{wc}(strong) \sim \frac{\sigma'_{wr}}{\sigma_{wr}} \cdot f_{wc}(feeble) \qquad (8)$$

A simplified estimation of the first-cracking stress σ_{wr} may be based on the horizontal strains of masonry.

a) In the case of zero-bonding (Fig. 6a):
- Horizontal strain of masonry $\nu_w\,\sigma_{wr} : E_w$
- Horizontal extensibility of mortar $f_{mt} : E_m$
- Compatibility of strains $\nu_w\,\sigma_w : E_w = f_{mt} : E_m$

Consequently $$\sigma_{wr} = \frac{1}{\nu_w} \cdot \frac{E_w}{E_m} \cdot f_{mt} \qquad (9)$$

b) In the case of a given bonding (Fig. 6b).
- Force resisting vertical cracking

$$F = f_{mt} H + \frac{H}{h_b} \cdot \mu \sigma'_{wr} \cdot c$$

where "c" denotes an average overlapping length
(and, since $\delta_{bg} = 2\frac{c}{l_b}$, it will be $c = \frac{\delta_{bg}}{2} \cdot l_b$),
whereas "μ" is a friction coefficient.

– Nominal increase of tensile strength of the vertical joints

$$F : H = f_{mt} + \mu \frac{c}{h_b} \cdot \sigma'_{wr}$$

– Compatibility of strains

$$\frac{1}{E_m}\left(f_{mt} + \mu \frac{c}{h_b} \cdot \sigma'_{wr} \right) = v_m \cdot \frac{\sigma'_{wr}}{E_w}$$

or $$\sigma'_{wr} = \sigma_{wr} : \left(1 - \frac{\mu}{v_m} \cdot \frac{E_w}{E_m} \cdot \frac{c}{h_c} \right) \qquad (10)$$

The ratio of the two cracking stresses may now be written as

$$k_2 = \frac{\sigma_{wr}}{\sigma'_{wr}} = \left[1 - \frac{1}{2}\left(\frac{\mu}{v_m} \cdot \frac{E_w}{E_m} \cdot \frac{l_b}{h_b} \right) \cdot \delta_{bg} \right] \qquad (11)$$

or

$$\left. \begin{aligned} & k_2 = 1 - \frac{1}{2}\gamma \cdot \delta_{bg} \\ & \text{where } \gamma = \frac{\mu}{v_m} \cdot \frac{E_w}{E_m} \cdot \frac{l_b}{h_b} \end{aligned} \right\} \qquad (11a)$$

Now, following the assumption of Equ. 8, $f_{wc,\delta=0} = k_2 \cdot f_{wc,\delta=\delta}$. And considering that the values of strength "f_{wc}" estimated by means of Equ. 4 and Equ. 6 are valid for δ_{bg}=1, (i.e. for $k_2 = 1 - \frac{\gamma}{2}$), we may write

$$f_{wc,\delta_{bg}} = \frac{1 - \gamma / 2}{1 - \gamma\delta_{bg} / 2} \cdot f_{wc} \qquad , \left(0 \le \delta_{bg} < 1 \right) \qquad (12)$$

It is however admitted that this approach is oversimplified, and it was followed only as an attempt to identify the basic parameters governing the consequences of inadequate bonding. Thus, for the time being, the direct use of Equ. 11a is not recommended; it would give the impression of a fictitious precision. Instead, it may be suggested to use an admittedly rough expression assuming γ ~ 1, valid for average conditions, with a very large scattering though.

Thus, $k_2 \sim 1 - \frac{\delta_{bg}}{2}$, and $f_{wc,\delta_{bg}} = f_{wc} : 2\left(1 - \frac{\delta_{bg}}{2} \right)$ (13)

2.6. Transversal bonding

Although two-leaf and three-leaf masonries are not the subject of this article, it is important to try to quantify (be it in a rather rough way) the structural consequences of transversal bonding between the two "faces" of a one-leaf masonry. Fig. 7 is a reminder of the vulnerability of transversally unbonded walls. Blocks protruding beyond the vertical axis of the transversal cross-section of the wall, contribute to the structural collaboration of its two "faces". That is why a "transversal bonding factor" could be defined[3] as follows (see Fig. 8)

$$\delta_{tr} = \left(\sum l_i\right) : H, \qquad 0 < \delta_{tr} < 1^{(4)} \qquad (14)$$

Attempting to quantify the structural consequences of inadequate transversal bonding, the same approach as in §2.5 may be followed: Values of the vertical stresses "σ_w" will be sought initiating internal quasi-vertical cracking near the axis of the transversal cross-section. A clearly oversimplified approach will be followed here, its scope being merely to identify the governing parameters only.
To this end, it will be assumed that (accidentally or because of internal inhomogeneities) the entire load $\sigma_w \cdot b_w$ is eccentrically acting only on the one half of the width of the wall, generating a differential shortening along the vertical axis of the transversal cross-section

$$\varepsilon = \zeta \cdot 2\sigma_w : E_w$$

where "ζ" is a variable numerical factor.
Corresponding maximum shear deformation along the "vulnerable" length $H\left(1-\delta_{tr}\right)$:

$$s_{crit} = \zeta \frac{2\sigma_w}{E_w} H\left(1-\delta_{tr}\right)$$

At this critical value of slip (s_{crit}), the frictional dilatancy exceeds the lateral deformability of the wall, and a vertical internal crack is produced, leading to a rapid destabilisation of the wall. The acting vertical compressive stress "σ_{wr}" at this stage is taken as a pre-estimator of the compressive strength $f_{wc,\delta_{tr}}$ of a wall disposing a transversal bonding factor δ_{tr}.
Critical situation

$$\sigma_{wr} \sim \left(\frac{s_{crit} \cdot E_w}{2\zeta \mathrm{H}}\right) : \left(1-\delta_{tr}\right)$$

Pseudo-quantitative estimation of

$$\sigma_{wr} \sim \sigma_0 : \left(1-\delta_{tr}\right) \qquad (15)$$

where $\sigma_0 = s_{crit} \cdot E_w : 2\zeta \mathrm{H}$, a constant.

As it was previously observed, in real walls, the maximum possible t_{tr}-value is close to 0,5, corresponding to a maximum possible value of $\sigma_{wr} = 2\sigma_0$.

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3 The necessary in situ measurements, however, are extremely difficult. Unless an intensive georadar investigation has taken place, information regarding the transversal cross-section of a masonry wall is collected only by means of local removal of stones, mainly near the sides of doors and windows.

4 In real masonry walls however, because of their practical bonding, the maximum value of t_{tr} is equal to 0,5.

Consequently, since σ_{wr}-values were considered as pre-estimates of f_{wc}-values, it may be written

$$f_{wc,\delta_{tr}} = \frac{1}{2\left(1-\delta_{tr}\right)} \cdot f_{wc} \quad , \qquad \left(0 \le \delta_{tr} \le 0,5\right) \qquad (16)$$

where “f_{wc}” denotes the compressive strength of fully bonded masonry (as those empirically predicted by Equ. 4 and Equ. 6).

Note 1
Nevertheless, in view of the immaturity of the state-of-the-art on the subject, instead of the aforementioned attempts to “calculate” the structural consequences of the bonding condition (plane and transversal), it seems more practical to try to match a given masonry wall, with one of the “models” shown in Fig. 9. Subsequently, appropriate strength-reduction factors will be applied, derived from experience or analogous laboratory tests. And this is the successful trend in the Italian Guide-lines.

Note 2
As it is well known, the problem of bidiagonal compression faces two complications. The first is related with the pronounced orthotropy of masonry, i.e. the considerably different structural properties along a vertical and a horizontal axis. And the second, is the biaxiality of stresses (diagonal compression, under a simultaneous perpendicular tension), which induce a considerable reduction of diagonal compression strength. This reduction is conventionally taken to be equal to 40%, but it very much depend on the form of the “critical” curve in the compression-tension region (a straight line in the Tresca-criterion, an ellipse in Von-Mises, or an outside-bulgy curve suggested by Page, and experimentally confirmed by this author).

Note 3
The presence of timber reinforcement within the body of masonry, offers a considerable increase both of strength and ductility, via mechanisms structurally similar with those governing the steel-reinforced masonry (see i.a. Psilla and Tassios: “Design models of reinforced masonry walls under monotonic and cyclic loading”, Eng. Structures, Dec. 2008).

Note 4
Two or three-leaf masonries were not included in this article. A recent state-of-the-art however may be found in Vintzileou E.: “Three-leaf masonry in compression, a review of literature”, Int. J. of Arch. Heritage, 5, 2011”).

3. COMPRESSIVE STRENGTH UNDER SEISMIC ACTIONS

Compressive actions on masonry under seismic conditions, may be distinguished as follows:
- Repetitive compression on intact masonry body.
- Repetitive compression on previously tensionally cracked masonry.
- Repetitive compression on previously diagonally cracked masonry.

Here again, the state-of-the-art is not rich. This article will be restricted in a short qualitative consideration of the first case only. In Fig. 10, when repetitively imposed "ε_{wc}" are smaller than $\varepsilon_{wc,u}$ (under monotonic loading), a relatively small stress-response-degradation (Δf_{wc}) is observed, accompanied by an increase ($\Delta\varepsilon_{wc,u}$) of the peak-strain $\varepsilon_{wc,u}$.
In Fig. 11, when repetitive actions start **after** static failure, an extremely rapid stress-response-degradation is observed, and the residual "strengths" σ'_{wc} may be negligible. Systematic numerical data on these subjects for various masonry types are missing…

4. DEFORMABILITY

4.1. Pseudo-elastic characteristics

a) Secant modulus of elasticity:[5]

$$E_w = \left[1 + \left(0,9\frac{\sigma_0}{f_{wc}} - 0,6\right)\cdot\frac{V}{V_{cr}}\right]\cdot E_{w0}$$

where

$E_w \sim (500 - 1500)\cdot f_{wc}$

σ_0, compressive stress acting on masonry (not higher than $\frac{2}{3}f_{wc}$)

V = acting shear force

$V_{cr} = b_w l_w \left(\frac{2}{3} f_{wt}\sqrt{1 + \sigma_0 / f_{wt}}\right)$

shear force at diagonal cracking

f_{wt}, horizontal tensile strength of masonry

$f_{wt} = \min(1,5\ f_{mt},\ \ \sigma_0,\ \ f_{bt}/2)$

(an extremely uncertain property)

b) Shear modulus:

Before cracking G_w = 0,3 to 0,4 E_{w0}

After cracking G_w = 0,2 E_{w0}

Angular distortion

– at starting of cracking

$$\gamma_{cr,0} = \frac{V_{cr}}{A_w} : G$$

– after visible cracking

$$\gamma_{cr} \sim 1,5\gamma_{cr,0}$$

4.2. Peak strain "ε_{wcu}" under compression

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5 Psilla, Tassios: "Design models of reinforced masonry walls under monotonic and cyclic loading", Eng. Structures, Dec. 2008.

a) This structural characteristic of masonry is of fundamental importance in the application of non-linear analysis of masonry structures. And although a certain ductile behaviour of these structures are of **systemic** origin, the ultimate strain of the material is very significant.

The values "ε_{wcu}" are **reduced** in the case of:
- higher strength mortar
- higher strength blocks
- better in-plane bonding
- better block/mortar adherence
- better completeness of joints
- transversal tension
- rapid loading

And they are **increased** in the case of:
- better transversal bonding
- previous small cracking
- compressive preloading
- available timber reinforcement
- perimetrically confined masonry
- lower performance-level (higher acceptable damage-level)

b) Numerical values under monotonic loading (data from NTUA):
- Rubble stone masonry
  - Peak value ε_{wcu} = 2 to 3.10^{-3}
  - After a 30% stress-response degradation
    ε_{wcu} = 3 to 5.10^{-3}
- Full-brick masonry (f_{wc} = 3 to 8 MPa)
  $\varepsilon_{wcu} = 4.10^{-3}$
- Hollow-brick masonry
  - Compression (f_{wc} = 2 to 3 MPa)
    ε_{wcu} = 1,5 to $3,5.10^{-3}$
  - Diagonal compression (f_{wc} = 3 to 4 MPa)
    ε_{wcu} =1 to 2.10^{-3}
- Timber reinforced masonries of all categories

$$\varepsilon'_{wcu} \sim 3 \cdot \varepsilon_{wcu}$$

c) Three-leaf masonry:
- Well-built external leafs and infill
  f_{wc} = 6 MPa, ε_{wcu} = 3 to 4.10^{-3} (Binda)
- Byzantine sub-normal masonry
  - before grouting (f_{wc} = 2,0 MPa)
    $\varepsilon_{wcu} = 1,5.10^{-3}$ (Vintzileou, Miltiadou)
  - after grouting (f_{wc} = 3,5 MPa)

$\varepsilon_{wcu} = 3{,}5.10^{-3}$

- Diagonal compression $\varepsilon_{wcu} = 1.10^{-3}$

In this respect, it is noted that certain non-linear analysis applications neglecting the aforementioned large variability of these critical strains, may not be in favour of safety and or economy. Nevertheless, it has to be admitted that systematic experimental and analytical investigations on this matter are also rather scarce…

Note

A rational method of estimating available rotational capacities of masonry piers or spandrel beams, may be found i.a. in T.P. Tassios: "Seismic Engineering of Monuments", The first Prof. N. Ambrazeys distinguished lecture, Bull. Earthquake Eng., 8/2010.

ADDENDUM A

In conclusion, unless an in situ or in-lab test of strength is carried out, the compressive strength of **stone-masonry** built with mortar, may be estimated (very roughly though) by means of the following empirical expression

$$f_{wc} = f_{wc0} : \psi_1 \cdot \psi_2 \cdot \psi_3$$

where

f_{wc0} ~ strength of well built, well bonded masonry, predicted by any reliable and calibrated empirical expression
(or, otherwise, $f_{wc0} = \frac{2}{3}\sqrt{f_b} + k_1 f_m - \sigma_0$, [MPa], Equ. 6)

ψ_1 ~ correction for thick mortar joints.
Perhaps $\psi_1 = 1 + 3,5\left(\frac{V_m}{V_w} - 0,3\right)$, Equ. 6

ψ_2 ~ correction for defective in-plane bonding.
Perhaps $\psi_2 = \frac{1 - \gamma\delta_{bg} / 2}{1 - \gamma / 2}$, Equ. 12

ψ_3 ~ correction for defective transversal bonding between the two faces of the masonry.
Perhaps $\psi_3 = 2\left(1 - \delta_{tr}\right)$, Equ. 16

ADDENDUM B

a) It is understood that verifications of the safety inequality in terms of **local** compressive stresses, should be carried out for stresses a v e r a g e d within a masonry volume compatible with the development of the local failure mechanism. The concept of such a “critical volume of failure” (CVF) is of basic importance: A compressive stress acting on a mathematical point is deprived of physical meaning; in any event, only the total force acting on a block (as well as the excentricity of this force will affect the structural behaviour of the block and its surroundings. The “mathematical” maximum stress should perhaps be compared with the strength of the block – not of the masonry!

b) Despite the fundamental importance (both for theory and design), this CVF-concept is not sufficiently discussed in literature.
A very short presentation of the concept follows hereafter:

(i) After the initiation of cracking (through mortar and/or block), the availability of **crack-arrestors** in the area, or a steep gradient of local stresses, may confine cracking within a small volume (V_1).

(ii) Despite the force-response degradation occurring in this area, stress **redistribution** in the surroundings may reinstate equilibrium (disturbance limited within a volume V_2)

(iii) After a further increase of external loading, crack-arrestors may be surpassed, damage may be spread and intensified, whereas redistribution of stresses is not anymore locally feasible. At this stage, the criterion of "acceptable level of damage" is used, in order to consider a loading situation as "ultimate".
The volume of damaged material at this stage is defined as "critical volume of failure"; within this volume, stresses may be averaged.

c) For the actual state-of-the-art however, in the case of masonry an approximate estimation of the size of a CVF, (for a "life protection" performance level), may indicatively be based on the dimensions of two consecutive blocks.

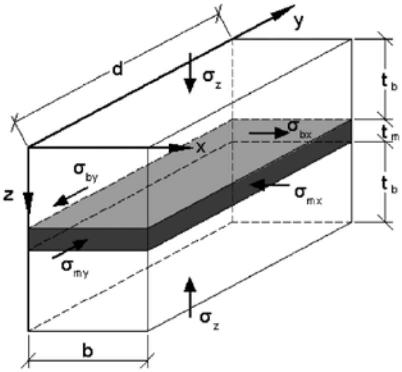

Fig. 1: Endogenous stresses in a masonry element under uniaxial external compression σ_z

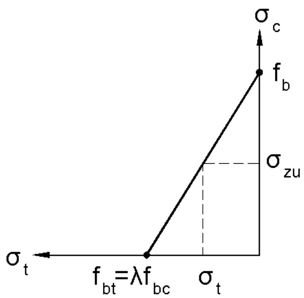

Fig. 2: Critical combination of compressive and tensile stresses acting on the block

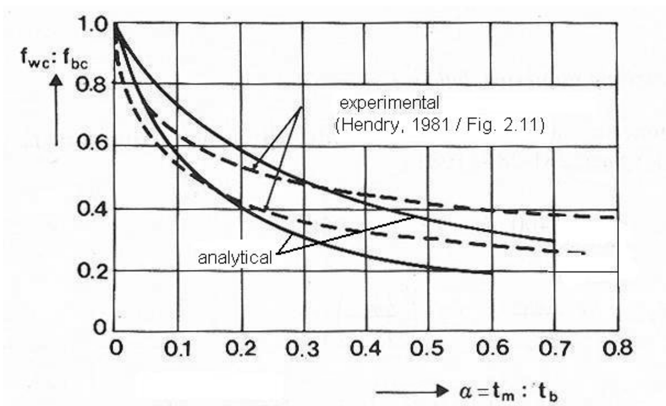

Fig. 3: Reduction of compressive strength of brick masonry, due to larger widths of mortar joints

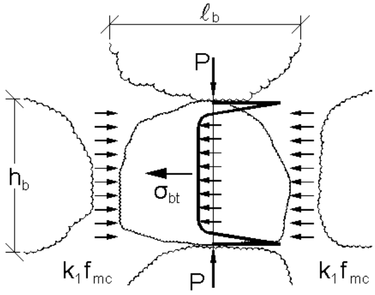

Fig. 4: In extremis, a rubble stone may be seated on its supporting stone on one point only

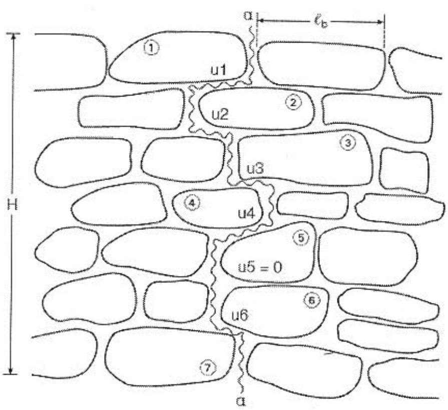

Fig. 5: Bonding may be quantified by summing up the overlapping lengths ‘u_i’ along a quasi-vertical section α-α

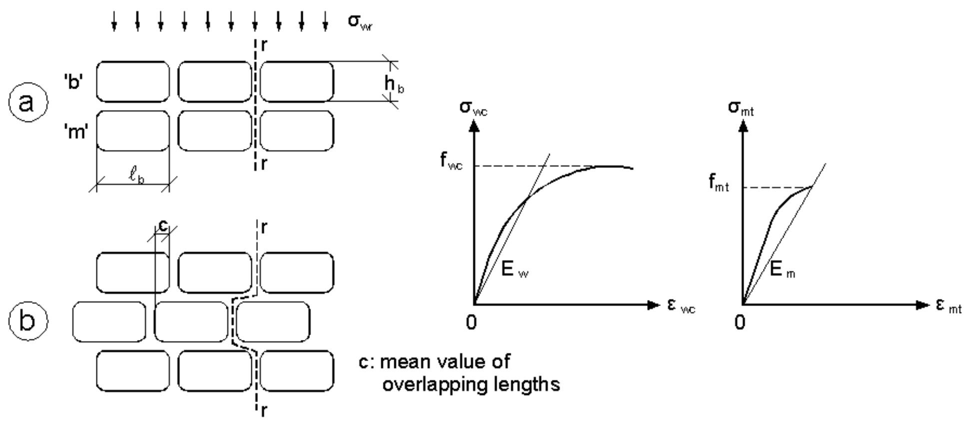

Fig. 6: Generation of a vertical crack r-r within mortar, due to the incompatibility of horizontal strains

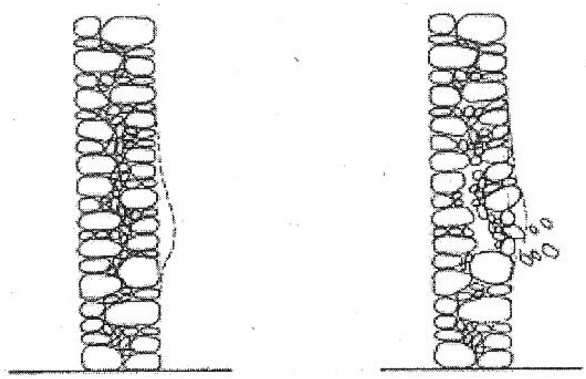

Fig. 7: Possible bulging on unbonded ‘faces’ of masonry (A. Giuffré)

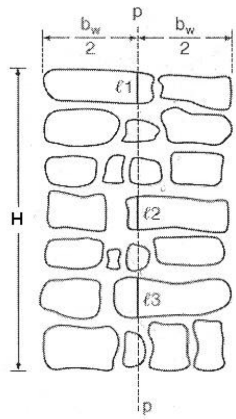

Fig. 8: The vertical axis p-p of the transversal cross-section intersects only relatively long blocks (lengths ℓ_1, ℓ_2, ℓ_3)

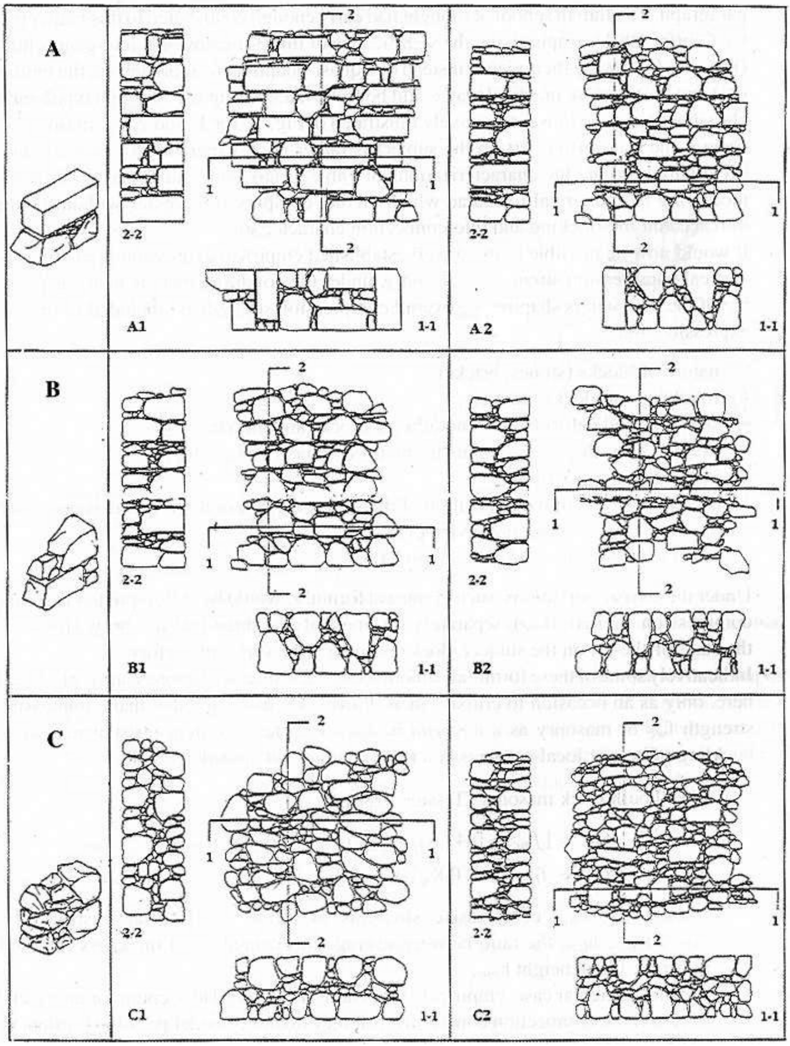

Fig. 9: Bonding models useful to ‘calibrate’ a given masonry wall and ‘classify’ it in a suitable ‘quality class’ A, B or C. (Italian Guidelines)

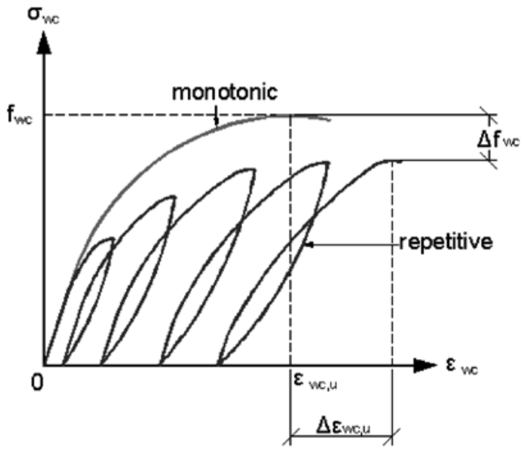

Fig. 10: Repetitive compression applied since the beginning of loading

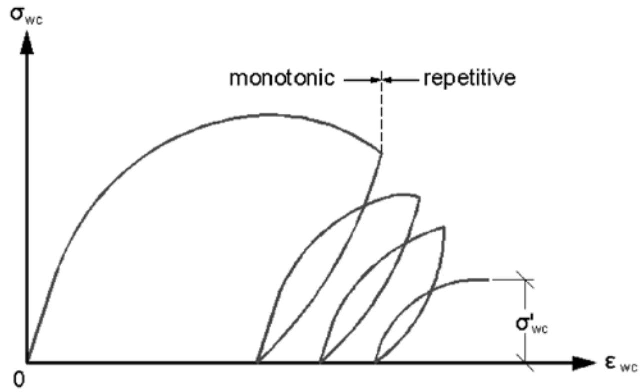

Fig. 11: Repetitive compression applied after the failure under monotonic loading