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ORIGINAL ARTICLE

# Crack width prediction of FRC beams in short and long term bending condition

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**Abstract** The use of short fibers inside concrete matrix is an effective method for reducing the vulnerability of concrete constructions subjected to harsh environment. The action of the short fibers in reducing the crack opening is the main issue that needs a research effort in order to optimize the expected results. At the moment the analytical prediction of the crack width and spacing in fiber reinforced concrete (FRC) structural elements under bending loads is still an open problem. A crack width relationship for FRC/RC elements similar to those developed for plain concrete structural members would be desirable for designers and engineers involved in the design of FRC structural elements. The recent development of important technical design codes, such as RILEM TC 162 TDF and the new Model Code (MC) 2010, embrace this idea. However further validation of these models by experimental results is still needed. On the other hand the study of the influence of a sustained load on crack width in presence of a short fibers reinforcement is a topic almost unexplored and important at the same time. In this research the cracking behaviour of full-scale concrete beams reinforced with both traditional steel bars and short fibers has been analyzed under short and long term bending condition. A theoretical prediction of crack width and crack spacing was carried out according to international design provisions based on different analytical models. The theoretical results are discussed and compared in order to highlight the differences between the available models and to check the reliability of the theoretical predictions on the basis of the experimental data. A modified relationship to take into account of the presence of stirrups has been proposed on the basis of experimental results; furthermore, some critical aspects, such as the influence of the type of fibers and the effect of loading-time, have been underlined that should be addressed in future research work.

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**Keywords** Fiber reinforced concrete · Cracking · Service life

**Variables**

$f_{Fts}$ FRC constant stress value assumed over the tension part of the cross-section

$f_{eq1}$ Post-cracking strength obtained by tests performed on notched beams in four point bending condition according to UNI 11039

$w_{max}$ Maximum crack width

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| | |
|---|---|
| $w_m$ | Average value of crack width |
| $s_{r,max}$ | Maximum crack spacing |
| $s_{r,m}$ | Average crack spacing |
| $\varepsilon_{sm}$ | Average strains of the steel bars in tension |
| $\varepsilon_{cm}$ | Average strains of the concrete in tension |
| $\varepsilon_{cs}$ | Strain of the concrete due to free shrinkage |
| $E_s$ | Steel modulus of elasticity |
| $E_c$ | Concrete modulus of elasticity |
| $A_s$ | Area of the longitudinal reinforcement |
| $A_{ce}$ | Effective area of the concrete in tension |
| $\sigma_s$ | Stress in the tensile reinforcement calculated in a cracked section under the applied external load |
| $c$ | Concrete cover thickness |
| $\phi_s$ | Bar diameter |
| $\phi$ | Diameter of the fiber |
| $L$ | Length of the fiber |
| $w_b$ | Maximum crack width at the bottom of the beam |
| $w_s$ | Maximum crack width at level of reinforcement |
| $\sigma_{sr}$ | Maximum steel stress in a cracked section at the crack formation stage |
| $A$ | The area of concrete symmetric with reinforcing steel divided by number of bars |
| $f_s$ | Is the reinforcing steel stress |
| $t_b$ | The bottom cover to center of bar |
| $t_s$ | The side cover to center of bar |
| $h_1$ | Is the distance from neutral axis to the reinforcing steel |
| $l_{smax}$ | The length over which slip between concrete and steel occurs |
| $w_k$ | Maximum crack width |
| $\tau_{bm}$ | The mean bond strength between steel and concrete |
| $\varepsilon_{sh}$ | Is the shrinkage strain |

## 1 Introduction

Cracking in concrete is related to its limited tensile deformation capacity and can be usually observed under service loads. The presence of cracks may significantly influence not only the esthetic of the structure but also its durability, permitting water and contaminants to enter, causing corrosion of reinforcing steel accompanied by a deterioration of concrete. In this contest, the ability of fibers in restraining crack widths of concrete structural elements, can be conveniently exploited to improve the durability of building and infrastructures and therefore the constructions sustainability.

The effectiveness of the fibers in reducing cracking in concrete structural elements depends on several factors, such as the type of fibers, their geometry, their amount, as well as on parameters that normally influence the cracking phenomena in reinforced concrete elements, namely the reinforcement ratio, the concrete cover thickness, the presence of stirrups, the bars diameter, the bars spacing, etc. The mixing sequence, seems also playing a significant role in determining the final properties of the FRC materials [1]. A large number of studies published in the last years testify that research interest in the field is very high. The use of non-traditional fibers as well as new technological solutions were even investigated. For instance recent researches were focused on the behavior of FRC engineered by using natural fibers [2] or on flexural behaviour of FRC when a cold plasma superficial treatment of the polymeric fibers is applied [3]. It was found that proper plasma treatment conditions significantly promote a multiple cracking of fiber reinforced concrete elements. Even if different aspects concerning the FRC performances were studied [4–7] as well as emerging solutions investigated, the analytical prediction of crack width and spacing in FRC/RC elements in bending is still an open problem, recently studied in [8] for FRC used in tunnel lining. In fact, to date there are not widely accepted relationships able to predict crack widths in presence of short fibers; this lack is still an obstacle to a widespread application of short fiber as crack-controlling reinforcement. According to Borosnyói and Balázs [9] the cracking process and the influence of fibers on the cracking development may be analyzed at different levels of accuracy related basically to four main approaches, namely analytical, semi-analytical, empirical and numerical.

For practical uses and design purpose, the second or third approach are normally considered; in fact available Codes, as the Model Code [10], the Eurocode [11] and ACI 224 R [12], join this kinds of approaches. As concerns the crack width predictions of FRC/RC (Reinforced Concrete), it would be highly desirable to provide design equations formally similar to those used for plain concrete, making the design approach much easier. The RILEM TC 162 TDF [13] and the new MC2010 [10] strongly embrace this idea. The RILEM TC 162 TDF provisions are based on the

European pre-standard ENV 1992-1-1 [11] while the crack spacing expression is modified in order to take into account the presence of fibers; in particular the aspect ratio of the fibers is considered as the most important variable involved in the problem. The MC 2010 provisions also modify the crack spacing formulation to include the influence of fibers; the effect of the residual load after cracking due to the presence of fibers is taken into account when considering the bond between concrete and steel bars.

The aforementioned design models need to be further validated by experimental studies. In fact, to date, there are few works in the literature which quantitatively relate the cracking pattern (width and spacing) to fiber dosage and fiber aspect ratio [14–18]. Most of the studies are focused mainly on the influence of fibers on the ultimate strength and stiffness of RC structural members. Even more the influence of a sustained load on crack width in presence of the fiber reinforcement, which is studied herein, remains a topic almost unexplored in literature. In general, the effect of time on crack width is a complex phenomenon which depends on several factors, such as the variability of concrete properties, the possible variation of water content, temperature and loading level; normally simplified methods are introduced in the codes for analyzing the effect of time on crack widths [10, 11]. The simplified approaches provided for design are in good accordance with laboratory tests performed on plain concrete structural element, but in case of FRC their effectiveness should be still verified. In fact, the presence of fibers may affect both the stress dependent and stress independent component of the long term concrete strains. While the effect of the presence of fibers on concrete shrinkage has been investigated by different Authors [19–21], also in presence of shrinkage reducing admixture [22], very few studies were devoted to the effect of long term loading on crack width of FRC structural element. To the best knowledge of the Authors only the works of Tan et al. [15, 23] focuses on this topic, thus further experimental research is needed in the field.

In the present study, the crack width and crack spacing relationships of RILEM TC 162 TDF and MC2010, together with ACI 224 and EC2 provisions, have been utilized referring to RC/FRC full scale beams, reinforced with steel or polyester fibers; the beams were tested under short and long term bending condition and their cracking pattern, namely crack width and crack spacing, was accurately registered during the tests at regular intervals. The theoretical predictions of crack width and spacing, carried out according to the available codes, were compared each other in order to evidence the main differences and the influence of empirical coefficients introduced by the different codes. They were also compared with experimental results in order to validate or propose modification to the available formulae.

## 2 Experimental program

Two sets of beams, S1 and S2, were designed and poured: S1 beams were used for long term bending test while S2 beams were tested under monotonic load (short term bending test).

One year after casting, the S1 beams were positioned under two loading steel frames; five beams per frame were piled up and loaded by means of a screw jack (Fig. 1a, b).The supports position was designed in order to counterbalance the self-weight of each beam (Fig. 1b). In order to simulate a loading service condition, a sustained load equal to 50 kN, about 50 % of designed ultimate load, was applied. Two loading cells were placed under the screw jacks to monitor the applied load periodically. An electronic acquisition system was used to check possible load variations; therefore when relaxation effects of the system caused undesired load drops, the beams were reloaded up to attain the initial value of 50 kN. The beams of frame 1 were unloaded after 17 months for laboratory testing, while the beams of frame 2 are still under load. The results of laboratory bending tests and durability tests carried out on beams of frame 1 are widely detailed and commented in [24].

During the 17 months of loading, measurements on the cracking pattern, namely crack width, crack length and crack position were carried out, periodically. Up to 9 months, crack widths were measured by means of an optical scale loupe, with a precision of 0.05 mm. Afterwards an hand held digital microscope with 200× magnification and higher resolution was used. The crack widths were measured along each beams between the loading points, at the bottom of their tension side.

S2 beams were cast in laboratory and tested under a four point bending scheme up to failure after about 2 months from casting. Deflections at mid-span and at

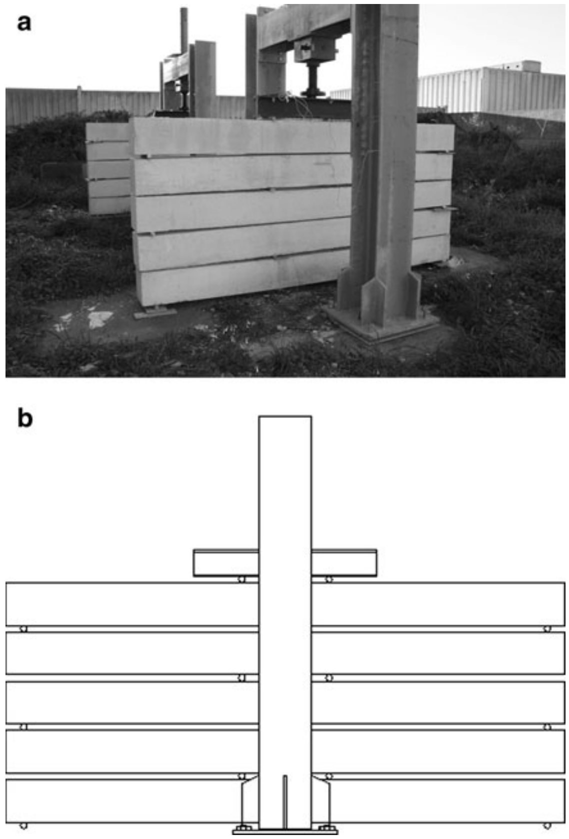

**Fig. 1** **a** S1 beams under sustained loading. **b** Details of supports position

quarter points of the beams were measured during the test by means of three resistive displacement transducers with a stroke of 50 mm. The cracking pattern of each beam was accurately analyzed at five load steps: step 1: 20 kN; step 2: 30 kN; step 3: 50 kN; step 4: 80 kN; step 5: 100 kN. At each load step, the number of cracks, the crack widths and the crack lengths were registered. As for S1 beams, the crack widths were measured at the bottom of their tension side by means of a digital microscope with a magnification up to 200×.

A brief summary of beams typologies and their loading history is reported in Table 1.

### 2.1 Materials

Three different concrete mixes were prepared to realize S1 and S2 beams: a control mix without fiber reinforcement (TQ), a concrete mix embedding steel fibers with a 0.6 % volume dosage (ST) and a concrete mix embedding polyester fibers with a 0.9 % volume dosage (POL). The geometrical and mechanical properties of the fibers are summarized in Table 2. All the mixes had a water/cement ratio equal to 0.65, a cement type 32.5R II-A/LL and a workability class S5 which means superfluid concrete (Table 3) [25]. Four cubes (150 mm side) for each mix (TQ, ST and POL) were cast for quality control. In Table 4, the values of the compressive strength obtained after 28 days from casting are reported. Fracture tests, fully described in [26], were performed on notched beams. In Fig. 2, the results of the tests are shown in terms of load-crack tip opening displacement curves. It can be observed that, as expected, steel fibers were more effective than polyester fibers in improving the toughness of the matrix.

In Table 5, the mechanical properties of longitudinal bars and stirrups employed in the beams, determined following UNI EN ISO 15630-1 [27] (three samples for each diameter), are reported. The nomenclature of the S1 and S2 beams is given in Table 6: as before mentioned, the code TQ refers to beams realized with plain concrete, while the codes ST and POL refers to FRC beams embedding steel and polyester fibers, respectively; the suffix _E is related to S1 beams.

### 2.2 Beams details

Rectangular reinforced concrete beams were designed according to Italian Code [28] based on EC-2, and following the loading scheme shown in Fig. 3: it is a four point bending scheme with a 2,800 mm span length and a 900 mm distance between the two loading points. The specimen sizes were decided in order to obtain a swallow beam without shear deficiency since a flexural failure was desired. The section geometry 250 × 250 mm and the length were adopted to allow an easy handling of the ten beams in laboratory and during transportation. The amount of steel reinforcement was calculated in order to have a ductile bending failure of the beam, with concrete crushing after steel yielding. Vertical stirrups were also provided to prevent premature shear failure, in accordance with the design code. The stirrups spacing, which is narrower along the regions close to supports, was applied in order to simulate a real situation in the field. Figure 3 shows the geometry of the beams and bars details. Three 14 mm diameter bars were placed

**Table 1** Summary of beam typologies and their loading history

| Beams | | History |
|---|---|---|
| S1 | Long term bending test frame 1 | Cast in place |
| | | One year of curing |
| | | Exposed to weathering under load for 17 months |
| | | Tested in bending up to failure |
| | Long term bending test frame 2 | Cast in place |
| | | One year of curing |
| | | Still exposed to weathering under load |
| S2 | Notexposedbeams | Cast in laboratory |
| | | 2 months of curing |
| | | Tested in bending up to failure |

at the tension region and two 14 mm diameter bars were placed at the upper compression region of the beams longitudinal reinforcement, while 8 mm diameter stirrups were placed at 14 cm over the entire length of the beam, except near the supports where the spacing was 7 cm.

## 3 Experimental results and discussion

### 3.1 Long term bending test

Position, width and length of the cracks of each S1 beam were registered during the loading period. The results of the measurements are extensively reported and commented in [29]. In Figs. 4 and 5 the average crack width measured between the two loading points of each beam under frames 1 and 2, respectively, are reported versus time. The crack width values measured on FRC beams were lower than those measured on plain concrete beams (TQ). This effect increased with the loading time. In fact, the crack widths of FRC beams (ST and POL) seems to be stabilized after 10 months of exposure, while those of TQ beams continued to grow until the last measurement. In Figs. 6 and 7, the average of crack widths measured after 17 months of loading, by using the electronic-optical apparatus, along the constant moment portion of each beam of frame 1 and 2, respectively, are shown. As can be noticed from the graphs, fibers caused an higher scatter of results compared to plain

**Table 3** Concrete mix

| | ST | POL | TQ |
|---|---|---|---|
| CEM 32.5R II-A/LL (kg/m$^3$) | 300 | 300 | 300 |
| Superplasticizer CRTV-L (kg/m$^3$) | 1.59 | 2.50 | 1.77 |
| Sand (0–4) (kg/m$^3$) | 1022.7 | 1030.1 | 1044.1 |
| Gravel (4–10) (kg/m$^3$) | 703.3 | 705.5 | 715.0 |

**Table 4** Cube (150 mm) compressive strength

| | Beam | Cube strength (MPa) | COV (%) |
|---|---|---|---|
| S1 | TQ | 25.8 | 4 |
| | ST | 21.4 | 8 |
| | POL | 23.2 | 8 |
| S2 | TQ | 22.70 | 6 |
| | ST | 19.84 | 6 |
| | POL | 22.65 | 5 |

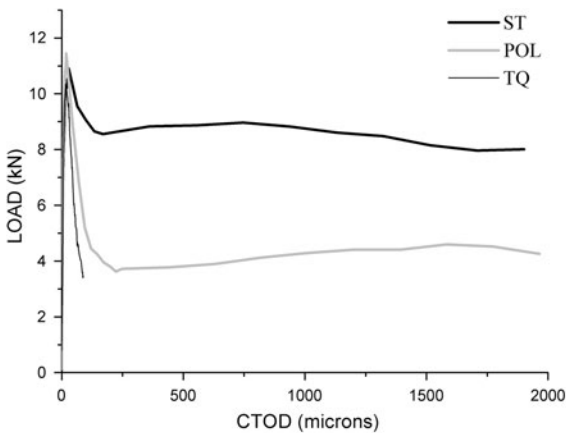


**Fig. 2** Fracture tests results

**Table 2** Geometrical and mechanical characteristics of the fibers

| Type | Shape | Diameter | Length | L/D | Tensile strength | Elastic modulus |
|---|---|---|---|---|---|---|
| Steel | Hooked | 600 μm | 30 mm | 50 | >1,150 N/mm$^2$ | 210 × 10$^3$ N/mm$^2$ |
| Polyester | Waved | 450 μm | 30 mm | 66 | 400–800 N/mm$^2$ | 11.3 × 10$^3$ N/mm$^2$ |

**Table 5** Mechanical properties of steel bars

| Materials | Diameter (mm) | Yield strength (MPa) | Ultimate strength (MPa) | Elongationatrupture (%) |
|---|---|---|---|---|
| Longitudinal bars | 14 | 520 | 614 | 12.2 |
| Stirrups | 8 | 567 | 600 | 4.8 |

**Table 6** Specimen labels

| Series | Beamtypology | Beam code |
|---|---|---|
| S1 | Exposed beams—frame 1 | TQ1_E, ST1_E, ST2_E, POL1_E, POL2_E |
| S1 | Exposed beams—frame 2 | TQ2_E, ST3_E, ST4_E, POL3_E, POL4_E |
| S2 | Notexposed beams | TQ1, TQ2, ST1, ST2, POL1, POL2 |

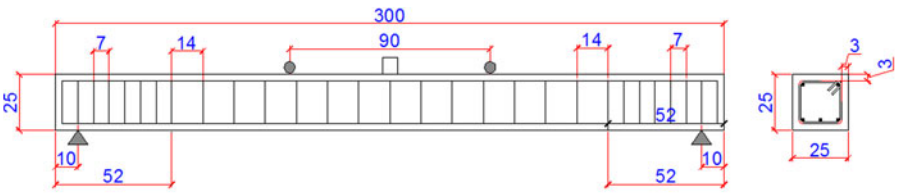


**Fig. 3** Loading scheme and beams details (dimensions in cm)

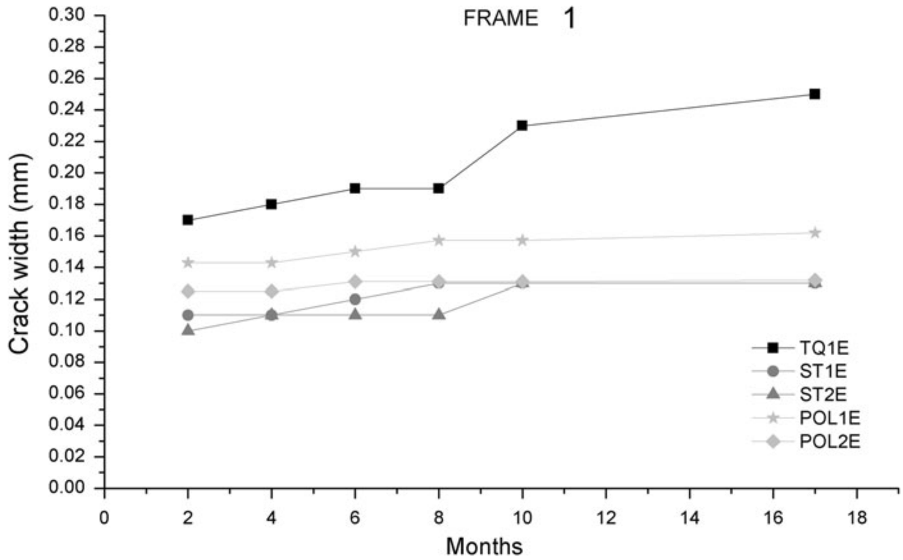


**Fig. 4** Average crack width versus time measured on S1 beams of frame 1

concrete beams. This is probably due to the effect of fiber dispersion in the mix and their variable orientation respect to crack direction, which caused some variability in the fiber effectiveness in restraining crack openings.

The number of cracks did not change during the period in which regular monitoring was performed. Figures 8 and 9 show the average crack spacing calculated between the loading points of each beam of frame 1 and 2, respectively. From a statistical analysis of variance (ANOVA) it results that, at 5 % level of significance, the presence of fibers did not influence the crack spacing values. The crack spacing values were strongly influenced by the spacing of stirrups, equal to 140 mm. The same result was found by Tan et al. [15]. This is due to the fact that stirrups generate a

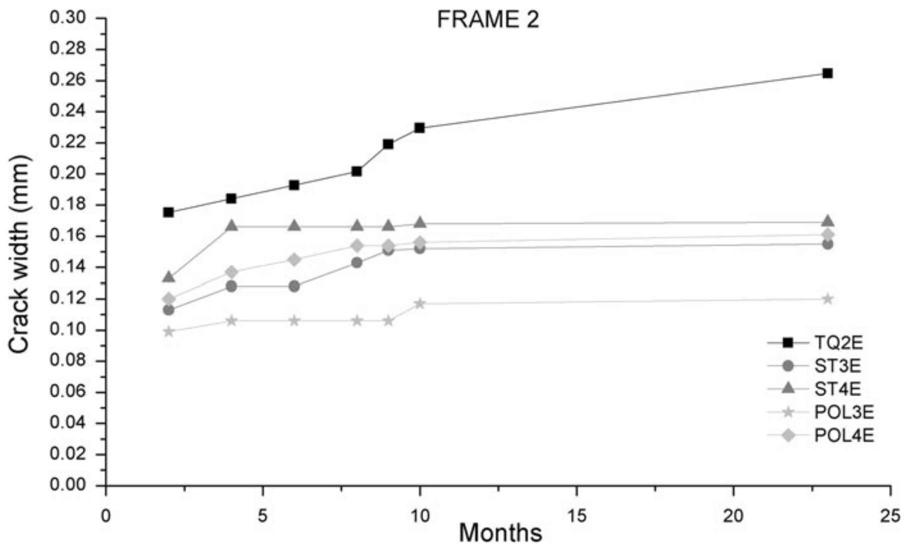


Fig. 5 Average crack width versus time measured on S1 beams of frame 2

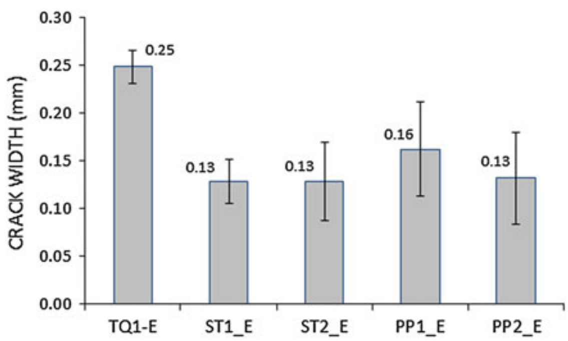


Fig. 6 Average crack width after 17 months of loading (frame1 S1 beams)

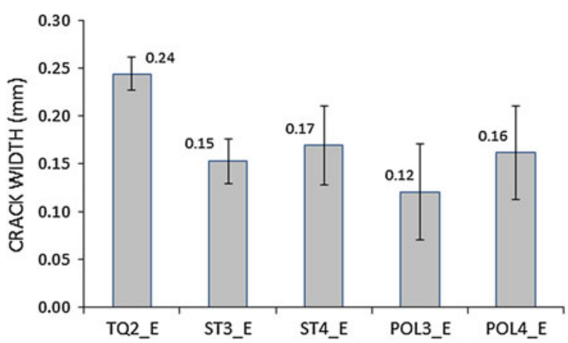


Fig. 7 Average crack width after 17 months of loading (frame2 S1 beams)

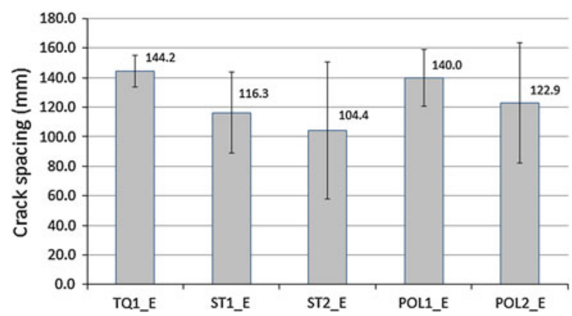


Fig. 8 Average crack spacing between loading points of S1beams of frame 1

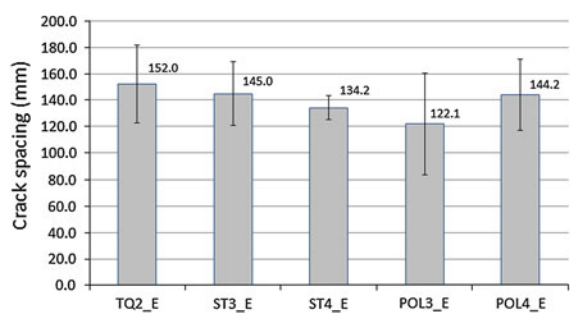


Fig. 9 Average crack spacing between loading points of S1beams of frame 2

natural discontinuity in the concrete, that represents a preferential nucleation region for cracks under tensile internal forces.

### 3.2 Short term bending tests

The results of tests carried out on S2 beams in bending are extensively described and commented in [21]. A summary of the cracking pattern results is reported herein.

Crack width measurements were made on S2 beams during the laboratory tests up to failure. In Fig. 10, the average crack widths calculated in the constant moment portion of the beams are reported for each load step investigated. In order to compare results of short and long term loading conditions, the average crack widths of S2 beams at step 3, equal to 50 kN, are considered in Fig. 11. It has to be noticed that the scatter of S2 beams results is higher than that of S1 beams at the same value of load. This effect is related to the progressive evolution of cracking with increasing load in S2 beams: in fact, while in the S1 beams the cracking pattern at 50 kN is completely developed (stabilized cracking) due to the long term loading, in the S2 beams new cracks were still growing at step 3. Thus the reported values of crack width for S2 beams is the mean value obtained referring to cracks formed in the previous stage of loading and those starting to open at 50 kN.

Comparing the long and short term crack width values, it can be noticed that the mean value of crack opening of TQ beams change from 0.20 mm in short term loading to 0.24 mm in long term loading, while the FRC crack width change slightly with the loading condition. Thus, the presence of fibers seems to reduce the crack growth with age respect to the behavior observed in plain concrete beams.

In Fig. 12, the average crack spacing calculated between the loading point of each S2 beam is reported.As for S1 beams, an high scatter of results can be observed: from a statistical analysis of variance (ANOVA) it results that, at 5 % level of significance, the presence of fibers did not influence the crack spacing values. As in the case of long term loading, the crack spacing is mainly affected by the presence of stirrups.

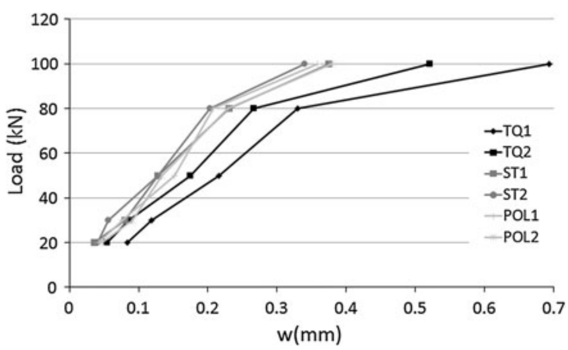


**Fig. 10** Load-average crack widths (*w*) curves of S2 beams

## 4 Code predictions for crack widths at serviceability limit state

An analytical prediction of crack width for S1 and S2 beams was performed on the basis of the recommendations available in Eurocode 2 [11], New MC 2010 [10], ACI 224 [12] and RILEM TC 162-TDF [13]. While the Eurocode2 and ACI 224 provisions refer only to plain concrete beams, the MC 2010 and

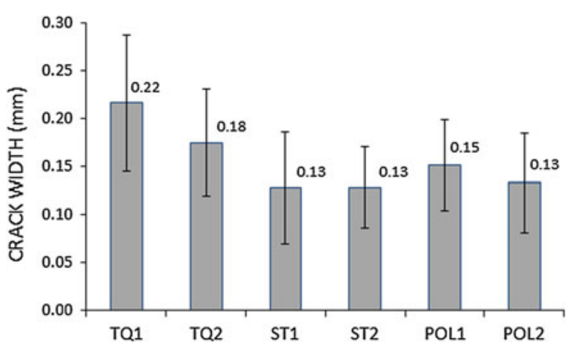


**Fig. 11** Average crack width of S2 beams at 50 kN

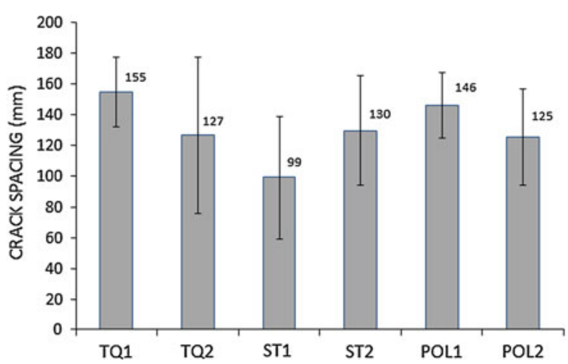


**Fig. 12** Average crack spacing of S2 beams

RILEM TC 162-TDF account for the presence of short fibers and consider a specific formulation for FRC structural members. The Eurocode2 and ACI 224 has been extended herein to the case of FRC taking into account the contribution of fibers in tension when evaluating the stresses distribution within the cracked cross section,. Specifically a constant stress ($f_{Fts}$) distribution over the tension part of the cross-section is adopted. Following Italian CNR DT-204-2006 guidelines [30], the value of $f_{Fts}$ is given by:

$$f_{Fts} = 0.45 f_{eq1} \tag{1}$$

where $f_{eq1}$ is the post-cracking strength obtained by tests performed on notched beams in four point bending condition according to UNI 11039 [31].

In the next paragraphs relationships provided by EC2, RILEM TC 162- TDF, MC2010 and ACI 224 to predict crack width and spacing under short and long term bending condition are summarized. All terms representing tension or compression in structural materials should be referred to N/mm$^2$.

### 4.1 Eurocode 2

According to EC2, the maximum crack width should be calculated as follows:

$$w_{max} = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) \tag{2}$$

where $s_{r,max}$ is the maximum crack spacing, and $\varepsilon_{sm}$ and $\varepsilon_{cm}$ are the average strains of the steel bars and the concrete in tension, respectively, over the length $s_{r,max}$. The maximum value of crack width is related to the average value ($w_m$) by the expression:

$$w_{max} = \beta w_m \tag{3}$$

where $\beta$ is a statistical coefficient equal to 1.7 [9, 32].

The difference between steel and concrete strains ($\varepsilon_{sm} - \varepsilon_{cm}$) in Eq. 2 is given by:

$$\varepsilon_{sm} - \varepsilon_{cm} = \sigma_s/E_s - k_t f_{ctm}/(E_s \rho_{s,eff})(1 + \rho_{s,eff}\alpha_e) \tag{4}$$

where $\alpha_e$ is the ratio between $E_s$(= steel modulus of elasticity) and $E_c$(= concrete modulus of elasticity); $\rho_{s,eff}$ is the ratio between $A_{s,}$ that is the whole area of the longitudinal reinforcement, and $A_{ce}$, that is the effective area of the concrete in tension. The value of $A_{ce}$ is obtained multiplying the width of the section (B) for $h_{c,eff}$, equal to the minimum value between 2.5($h - d$), ($h - x$)/3 and h/2 (Fig. 14). The coefficient $k_t$ is set equal to 0.6 for short-term loading condition and 0.4 for long term or cyclic loading; $\sigma_s$ is the stress in the tensile reinforcement calculated in a cracked section under the applied external load. In this work, the value of $\sigma_s$ for FRC beams has been calculated considering the contribution of the short fibers in tension, as before mentioned.

The crack spacing ($s_{r,max}$) has the following semi-empirical formulation:

$$s_{r,max} = k_3 c + k_1 k_2 k_4 \phi_s/\rho_{s,eff} \tag{5}$$

where $c$ is the concrete cover thickness (mm) and $\phi_s$ is the bar diameter (mm). The EC2 suggests to set $k_3$ equal to 3.4 and $k_4$ to 0.425; $k_1$ is a coefficient which accounts for the bond properties of steel bars (= 0.8 for corrugated bars and = 1.6 for smooth bars); $k_2$ is a coefficient which takes account of the form of strain distribution along the cross section (= 0.5 for bending and = 1 for pure tension).

In the case of long term loading the effect of shrinkage ($\varepsilon_{cs}$) must be taken into account when evaluating the concrete strain (Eq. 2).

### 4.2 Rilem tc 162-tdf

The RILEM TC 162-TDF proposes a crack width formulation which takes into account the presence of steel fibers. Starting from [32], the crack width is calculated according to:

$$\mathrm{w_k} = \beta \mathrm{s_{r,m}} \varepsilon_{sm} \tag{6}$$

in which $\beta$ is a coefficient (equal to 1.7 for load induced cracking)relating the average crack width to the maximum crack width; $s_{r,m}$ is the average crack spacing, which has a semi-empirical formulation:

$$\mathrm{s_{r,m}} = (50 + 0.25\, k_1 k_2 \phi_s/\rho_{s,eff})\,(k\phi/\mathrm{L}) \tag{7}$$

and $\varepsilon_{sm}$ is the average steel strain, which takes into account the additional contribute of concrete in tension (tension stiffening) and is expressed as:

$$\varepsilon_{sm} = \sigma_s/E_s[1 - \beta_1\beta_2(\sigma_{sr}/\sigma_s)^2] \tag{8}$$

where $k$ is set equal to 50, $k_1$ and $k_2$ have the same values already specified in Eq. (4); $\beta_1$ is a coefficient which takes account the bond properties of the bars (1 for corrugated bars and 0.5 for smooth bars); $\beta_2$ is a coefficient which takes account of the duration of the loading or of repeated loading (1 for single short term loading, 0.5 for sustained load or for many cycles of repeated loading); $\phi_s$ is the bar diameter (mm); $\rho_{s,eff}$ is the ratio between $A_s$, and $A_{ce}$. The calculation of $A_{ce}$ is that reported in EC2, assuming $h_{c,eff}$ equal to 2.5 ($h - d$) (Fig. 13). $\sigma_s$ is the stress in the tensile reinforcement calculated at the cracked section; $\sigma_{sr}$ is the stress in the tensile reinforcement calculated at a cracked section under loading conditions causing the first cracking; $L$ is the length of steel fiber (mm) and $\phi$ is its diameter (mm). For FRC,$\sigma_s$ and $\sigma_{sr}$ are determined taking into account the post cracking tensile strength of fiber reinforced concrete in the hypothesis of a constant stress ($f_{Fts}$) distribution over the tension part of the cross-section (Eq. 1).In the case of long term loading, the contribute of shrinkage has to be taken into account for the evaluation of $\varepsilon_{sm}$ (Eq. 8).

### 4.3 ACI 224R

The ACI approach is based on a statistical analysis of maximum crack width data from a number of sources. The equations that are considered to best predict the maximum bottom and side crack widths are:

$$w_b = 58.70\,(t_b A)^{1/3} \beta (f_s - 34.48) \times 10^{-3} \tag{9}$$

$$w_s = 58.70\,(t_b A)^{1/3} / (1 + t_s/h_1)\,(f_s - 34.48) \times 10^{-3} \tag{10}$$

where $w_b$ is the maximum crack width at the bottom of the beam (mm)while $w_s$ is that at level of reinforcement (mm); $f_s$ is the reinforcing steel stress (MPa); $A$ is the area of concrete symmetric with reinforcing steel divided by number of bars (mm$^2$); $t_b$ is the bottom cover to the center of bars (mm); $t_s$ is the side cover to the center of bar (mm); $\beta$ is the ratio of the distance between neutral axis and tension face to the distance between neutral axis and reinforcing steel, which is considered 1.20 in beams; $h_1$ is the distance from neutral axis to the reinforcing steel (mm). In order to adapt the ACI formulation to the case of FRC, the value of $f_s$ has been calculated here considering the contribute of fibers after cracking (constant stress $f_{Fts}$ over the tension portion of the cross section). In the ACI 224 R there is not an explicit formulation for long term loading. On the basis of experimental research from different sources, in the document is reported that a double value of crack width with time can be expected.

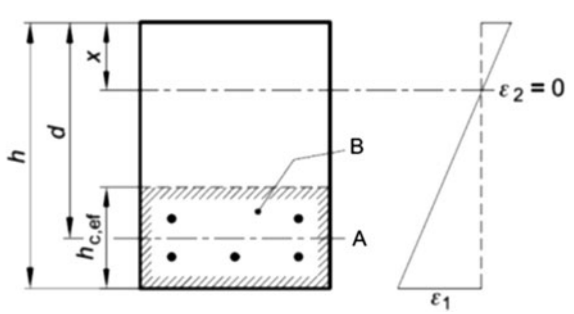


**Fig. 13** $A_{ce}$ for RC beams in bending according to EC2

Unlike the other codes, the ACI 224 R does not give an explicit formulation of crack spacing and average strain in the steel reinforcement and concrete.

### 4.4 Model code 2010

The new MC 2010 suggests two distinct formulations for plain and fiber reinforced concrete structural members. In both cases the maximum crack width can be calculated as:

$$w_k = 2 l_{smax} (\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs}) \tag{11}$$

where $l_{smax}$ is the length (mm) over which slip between concrete and steel occurs. $\varepsilon_{sm}$ and $\varepsilon_{cm}$ are the average strains of steel bars and concrete, respectively, over the length $l_{smax}$. $\varepsilon_{cs}$ is the strain of the concrete due to free shrinkage. The average crack width can be calculated by dividing the maximum crack width (Eq. 11) for 1.5 [9].

lsmax has two different expression for plain and fiber reinforced concrete (Eqs. 12 and 13, respectively):

$$l_{smax} = k\,c + f_{ctm} \phi_s / (4 \tau_{bm} \rho_{s,eff}) \tag{12}$$

$$l_{smax} = k\,c + (f_{ctm} - f_{tsm}) \phi_s / (4 \tau_{bm} \rho_{s,eff}) \tag{13}$$

where $f_{tsm}$ (Eq. 1) is the residual tensile strength of FRC equal to 0.45 $f_{R1}$ (similar to $f_{Fts}$ of CNR DT 204-2006 equal to 0.45$f_{eq(0-0.6)}$ [33]). The relative mean strain in Eq. (11) follows from:

$$\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs} = (\sigma_s - \beta \sigma_{sr})/E_s + \eta_r \varepsilon_{sh} \tag{14}$$

where $\sigma_s$ is the stress in the steel rebars at a cracked section, in which the effect of fibers needs to be taken into account; $\sigma_{sr}$ is the maximum steel stress in a cracked section at the crack formation stage, which is:

$$\sigma_{sr} = f_{ctm}(1 + \rho_{s,eff}\alpha_e)/\rho_{s,eff} \quad (15)$$

for plain concrete and:

$$\sigma_{sr} = (f_{ctm} - f_{tsm})(1 + \rho_{s,eff}\alpha_e)/\rho_{s,eff} \quad (16)$$

for FRC.

The values of $\tau_{bm}$ is equal to 1.8 $f_{ctm}$ for stabilized cracking in both short and long term loading. $\beta$ is equal to 0.6 and 0.4, for short and long term loading, respectively. $\eta_r$ is equal to 0 or 1, for short and long term loading, respectively. $\rho_{s,eff}$ and $\alpha_e$ are those reported in EC2 (4.1). Respect to the Model Code 1990 [34] the following parameter has been introduced:

$$(h - x)/(d - x) \quad (17)$$

that multiplying the $w_k$ value of Eq. (11) allows to calculate the crack width at the bottom of the tension side of the beams.

## 5 Comparison between the code predictions and experimental results

A comparison between the average crack width and spacing obtained experimentally and by means of the above mentioned codes was carried out.

### 5.1 Crack spacing

In Fig. 14, the values of the average crack spacing obtained experimentally and the theoretical values calculated according to EC-2, MC 2010, RILEM TC 162 and ACI 224 R design codes are reported. From the experiments it was observed that the crack spacing did not change with time (Sect. 3.1), thus the average experimental value used in the comparison was calculated as the mean value of the average crack spacing of S1 and S2 beams.

The crack spacing prediction obtained by RILEM TC 162 and MC 2010 relationships was found to be in good accordance with experimental results for TQ beams, while the EC2 formulation underestimates (21 %) the experimental values. The main difference between RILEM TC 162 and EC2 formulations lies in the $h_{c,eff}$ value that results lower in the EC2 (see Sects. 4.1 and 4.2).

A worse accordance with experimental results is obtained by MC2010 in the case of ST beams. In fact, the code's provision strongly underestimates (48 %) the experimental crack spacing. The MC 2010 differs from the other codes as it considers the residual tensile strength of the fibers ($f_{tsm}$) affecting the bond between

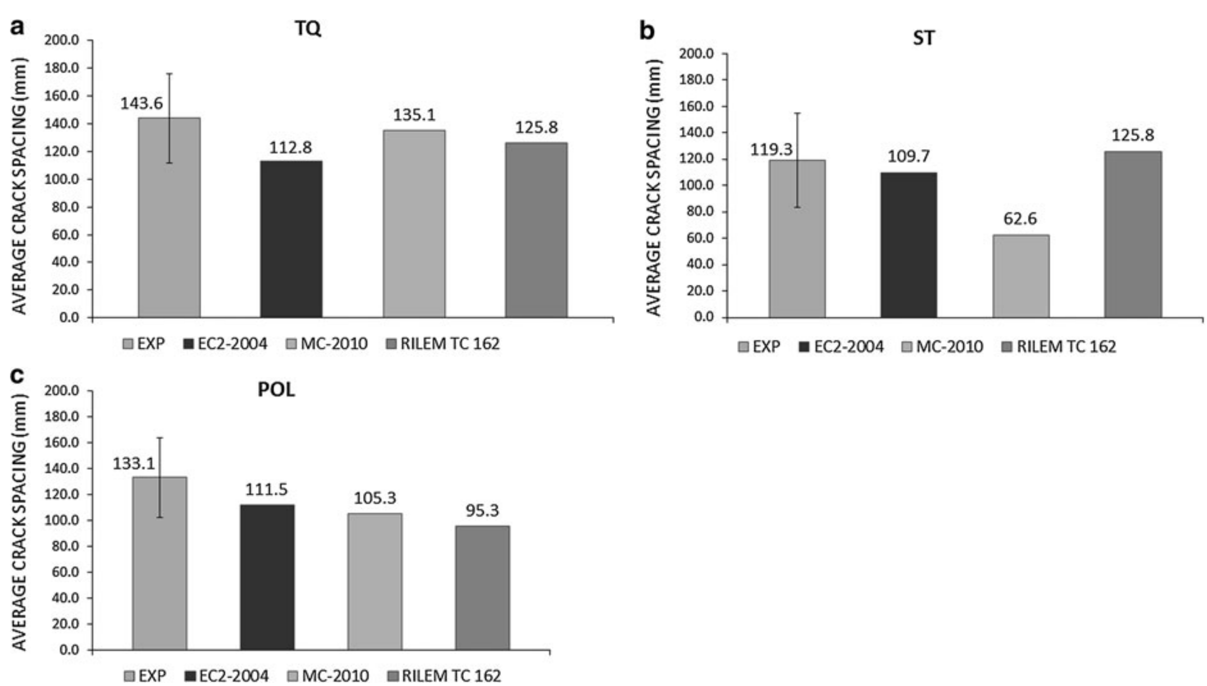


**Fig. 14** Comparison between experimental and theoretical average crack spacing of S1 and S2 beams (**a** TQ beams, **b** ST beams, **c** POL beams)

FRC and steel bars. This effect causes a lower value of the average crack spacing compared to the other formulations. It is in the opinion of the Authors that the poor accordance with the experimental results may be related to the presence of stirrups, which strongly influence the crack spacing of the tested ST beams. This hypothesis is proved by the application of MC 2010 formulae to the case study of Vandewalle [14], in which there were not stirrups between the loading points of the analyzed beams. In fact, referring to the case of the beams reinforced with 0.56 % volume dosage of steel fibers, the predicted average crack spacing results equal to 106 mm that is very similar to the experimental value (about 110 mm). Therefore a good accordance between the MC 2010 provisions and experimental results is found when the beam is not reinforced with stirrups between the loading points. For this reason a modification of the crack spacing formula is proposed (Eq. 18), based on the experimental result presented herein, to take into account of the presence of stirrups in the case of steel fiber reinforced beams:

$$s_{pm} = (2l_{smax}/1.5 + s_{st})/2 \quad (18)$$

where $s_{pm}$ is the proposed mean value of crack spacing; $l_{smax}$ refers to Eq. (13), $2l_{smax}$ is the maximum crack spacing and ($2l_{smax}/1.5$) is the average crack spacing according to MC 2010; $s_{st}$ is the spacing between stirrups. Applying the Eq. 18 to the tested beams, a value of 101.3 mm is obtained for cracks spacing, that is narrower to the experimental results (119.3 mm). Equation (18) can be applied also to TQ and POL beams, obtaining a crack spacing of 137.6 and 122.7 mm, respectively; both values are found to be in accordance with the experimental ones (143.6 and 133.1 mm for TQ and POL beams, respectively).

As regards RILEM TC 162 predictions, in the case of analyzed ST beams the product between $k$ and $\phi$/L in Eq. (7) is equal to 1, thus there are no differences in the relationship of crack spacing between ST and TQ beams; a good accordance with experimental results is still confirmed. More experimental research with other values of the fiber aspect ratio is needed to validate the RILEM TC 162 prediction, as well as the influence of stirrups.

A different result has been obtained in the case of POL beams since the product between $k$ and $\phi$/L is equal to 0.75. In this case an underestimation (28 %) of the crack spacing value with respect to the experimental one can be observed (Fig. 14c). Probably in case of fibers different from steel another value of $k$ (Eq. 7) have to be considered. Furthermore, more research is needed to evaluate the influence of the presence of stirrups on crack spacing.

### 5.2 Crack width: short term loading

In Fig. 15, the experimental average crack width obtained for S2 beams and those analytically evaluated by the code's equations are reported. Furthermore, the average crack width obtained applying the MC2010 formulation with an average crack spacing calculated according to Eq. 18 ("proposed model" in the graph) is also added. All the reported values refers to the crack opening at the bottom of the beams; thus for each code formulation the crack opening, calculated at the steel bars level, has been multiplied for the factor of Eq. 17.

It has to be underlined that the ACI 224 R provision refers only to the maximum crack width, thus the results of ACI 224 R reported in the graphs (Fig. 15) have to be compared to the maximum experimental value of crack width.

All the Codes investigated (included the proposed model) give a good prediction of experimental average crack width of TQ beams. As regards the ACI prediction, the value of crack width reported in Fig. 15a has to be compared with the maximum crack width obtained experimentally. The average of the maximum values of crack width obtained from the tests on beams TQ1 and TQ2 is equal to 0.28 mm, thus the prediction of ACI 224 R, equal to 0.27 mm, is good.

In the case of ST beams the EC2 relationship prediction multiplied by the factor of Eq. (17) furnishes a value of crack width equal to 0.12 mm that is in good agreement with experimental results. While the RILEM TC 162 TDF slightly overestimate the average crack width obtained experimentally remaining in any case within the scatter of experimental results. The average of the maximum value of crack width obtained from the beams ST1 and ST2 is equal to 0.21 mm, thus the prediction of maximum crack width of ACI 224 R equal to 0.20 is also enough satisfactory. As regards MC 2010, it strongly (30 %) underestimates the experimental crack width. This is mainly due to the low value of crack spacing obtained from Eq. (13), which does not consider the presence of stirrups. Using

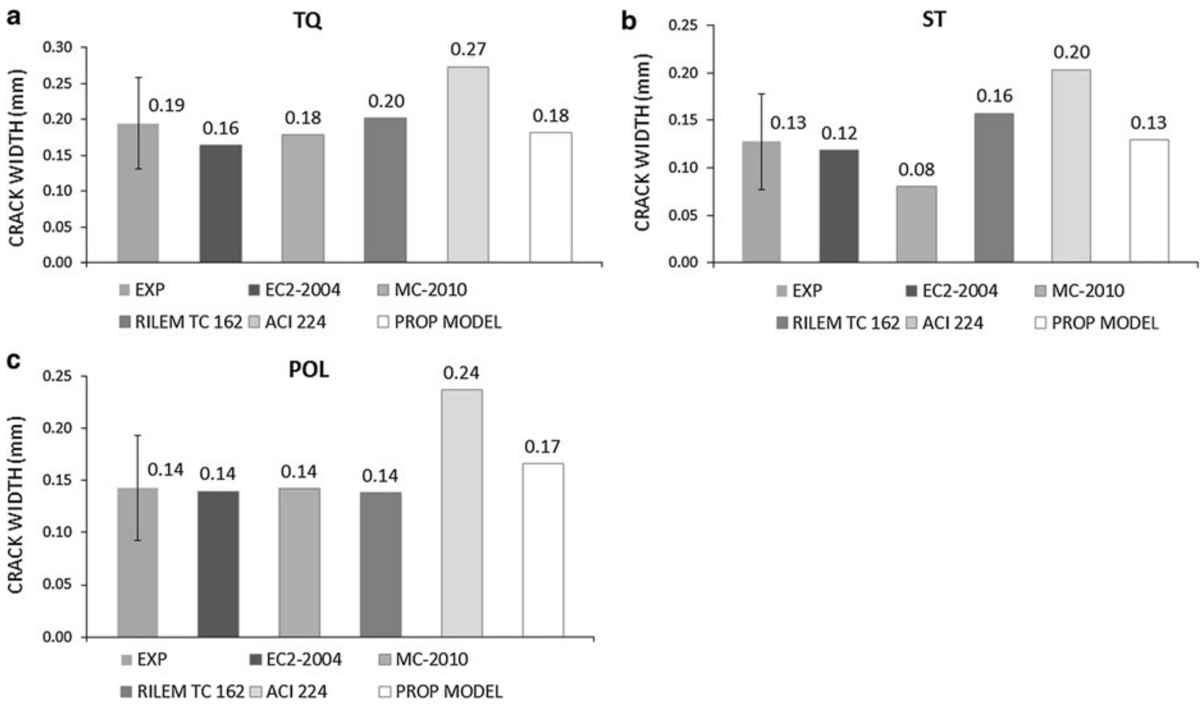


**Fig. 15** Experimental and theoretical crack width of S2 beams (short term loading) (**a** TQ beams, **b** ST beams, **c** POL beams)

the crack spacing value obtained from the proposed relationship (Eq. 18), a value of average crack width of 0.13 mm is obtained that is closer to the experimental results (Fig. 15b). If the MC 2010 formulae are applied to the case study of Vandewalle [14], considering a beam reinforced with 0.56 % volume dosage of steel fibers, a crack width prediction of 0.09 mm is obtained at the level of steel bars which is in good accordance with the experimental data(average crack width of 0.07 mm). This results further confirms that MC 2010 formulation should be modified for taking into account the influence of stirrups, the proposed revision seems effective when applied to the analyzed beams.

As regards POL beams, the MC 2010, EC2 and RILEM TC 162 predictions (0.14 mm) are in good accordance with the experimental value (0.14 mm), even if the average crack spacing value of the RILEM TC 162 underestimated the experimental value (Sect. 5.1).The average crack width of the proposed model slightly overestimates the experimental crack width remaining still in the scatter range of results. The average of the maximum values of crack widths obtained experimentally is equal to 0.18 mm which differs from the value predicted by the ACI 224 R formulation (0.24 mm).

### 5.3 Crack width: long term loading

In Fig. 16, the average crack width obtained from long term bending test and by codes provisions is reported. In particular the ACI 224 R is not considered for long term loading because it not gives a specific formulation to take into account the effect of time. On the other hand, in the graphs it is reported the crack width obtained applying the MC2010 formulation in which the average crack spacing given by Eq. 18 is considered (proposed model in the graphs). In the codes the delayed concrete strains due to the effect of time can be considered as the sum of two components: a stress-dependent strain and a stress-independent strain. In the case of temperatures between 20 and 40 °C, the stress independent component considered by the codes is equal to the free shrinkage of concrete. While to take account of the effect of loading with time a factor which multiply concrete strain is considered, that is $\beta$ in Eq. (14), $\beta_2$ in Eq. (8) and $k_t$ in Eq.(4).

In the present study the concrete strain due to free shrinkage has been calculated according to the codes considering a relative humidity equal to 78 % and a period of drying of 868 days.

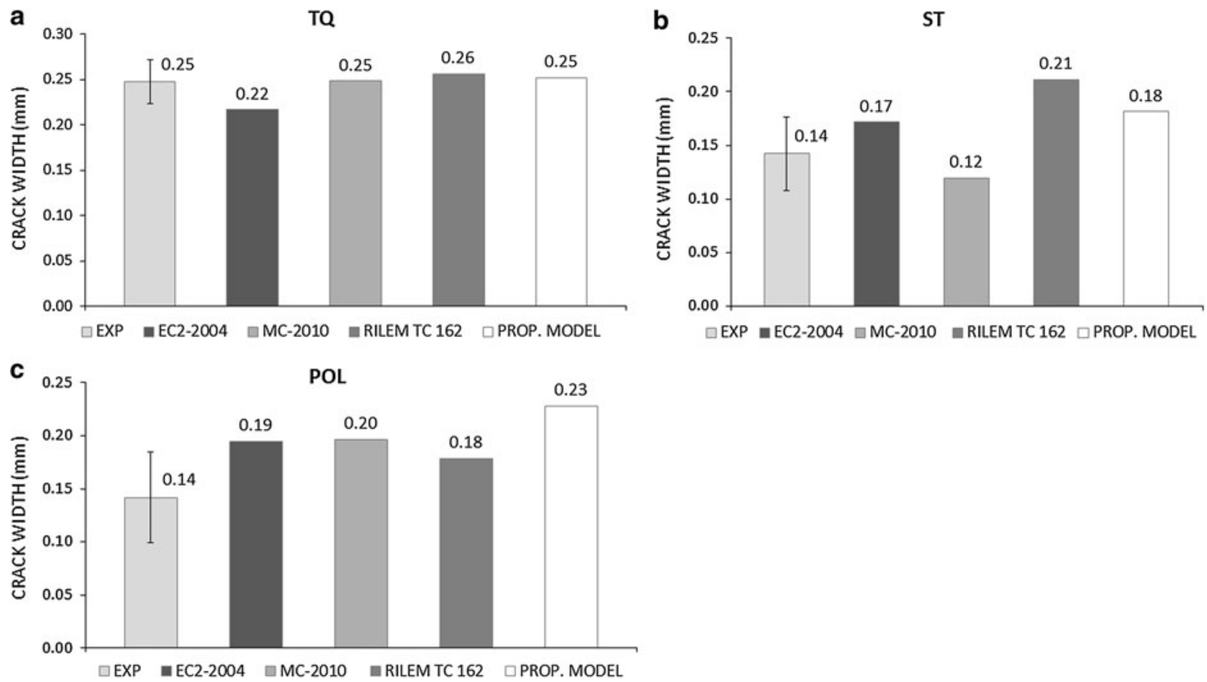


**Fig. 16** Experimental and theoretical crack width of S1 beams (long term loading) (**a** TQ beams, **b** ST beams, **c** POL beams)

Results of TQ beams showed that all the analytical models predict well the value of average crack width (also in the case of proposed model). In particular, the EC2 formulation slightly underestimate the average crack width obtained from experiments, as well as it was underlined in case of short term loading.

As regards ST and POL beams, a little increment in experimental crack width can be observed comparing long term and short term bending tests results, on the contrary for TQ beams the crack width increased from 0.19 mm of S2 beams (short term loading) to 0.25 mm for S1 beams (long term loading). In the case of TQ beams the codes predict well this increment in crack width due to the effect of time, but applying the same formula to the case of ST and POL beams, the effect of shrinkage and of loading on concrete strains appears overestimated. In Fig. 5 the value of average crack width predicted by the codes in short and long term bending condition are illustrated; the value of $\Delta_{\mathrm{time}}$ evidences the increment of crack width estimated by the codes due to the effect of time. The values of $\Delta_{\mathrm{time}}$ for all the codes are not in accordance with the experiments which evidenced a negligible increment in crack width due to the effect of time. The experimental results would evidence the significant contribution of fibers on long-term condition and, thus, on structural durability, not adequately considered by the present codes provisions. On the basis of this interesting results a wider experimental research is suggested to correctly quantify the effect of the presence of fibers on the free shrinkage and long term loading in the crack width prediction.

There are some works in the literature on the effect of fibers on drying shrinkage: the results are in some cases contradictory. Swamy et al. [21] found a reduction of free shrinkage of 21 and 14 % compared to plain concrete, with the addiction of 1 % straight steel fibers and crimped steel fibers, respectively. Barr et al. [35] showed that the addition of 2 % of crimped steel fibers with a length of 40 mm and a diameter of 0.75 mm reduced the free shrinkage by approximately 20 % for concrete with compressive strengths of 45 and 65 MPa, while the shrinkage of concrete with a strength of 30 MPa was unaffected. The favorable influence of fibers was contradicted by results presented in Altoubat and Lange [36] where no distinguishable effect of steel fibers on the free shrinkage was observed.

On the other hand to the best knowledge of the authors only the works of Tan et al. [10, 18] focuses on the effect of long term loading on crack width of RC beams with and without fibers, thus further investigations are strongly needed in order to reconsider the calibration of factors provided by available codes which account for the effect of long term loading.

## 6 Conclusions

An analytical prediction of the crack spacing and crack width of FRC/RC beams under bending loads has been carried out according to different codes, namely EC2, ACI 224 R, MC2010 and RILEM TC 162-TDF. The analytical results were compared each other in order to evidence the main differences between the codes provisions. Analytical results were also compared with experimental data in order to validate or propose modification to the design formulae.

- From the comparison between the theoretical predictions and experiments, it results that the average crack spacing given by design codes are in good accordance with experimental results referred to plain concrete beams, especially the MC 2010 provisions.
- In case of FRC beams, the EC2 prediction of crack spacing is in good agreement with experimental results even if a specific formulation for fiber reinforced concrete is not considered. In contrast, the MC 2010 prediction underestimates the experimental results especially in the case of ST beams. It is opinion of the Authors that this is due to the presence of stirrups which are not considered by the code but strongly influences the crack spacing in the experiments. A good prediction of crack spacing is obtained by a modified crack spacing relationship, proposed by the Authors, which takes into account the presence of stirrups. The RILEM TC 162 TDF prediction is in good accordance with the crack spacing obtained for ST beams for the specific analyzed aspect ratio, while it underestimates the crack spacing obtained for POL beams.
- The results of short term crack width prediction of MC 2010 are in good accordance with the experiments in the case of TQ and POL beams, while for ST beams a good prediction of crack width is obtained considering the effect of stirrups (proposed model) on crack spacing. The RILEM TC 162 TDF and the EC2 results are in accordance with the experiments when the increase in crack width at the bottom of the beam is considered. The ACI 224 R predicts well the maximum crack width of TQ and ST beams while it overestimate that of POL beams.
- Looking at results of the long term crack width, it has been found that all the codes are in good accordance with the experiments in case of TQ beams. In contrast, in presence of fibers (both steel and polyester) all the codes estimated an increment in crack width due to the effect of time which does not correspond to the experimental evidence.

An equation was proposed for prediction of crack spacing, calibrated with the experimental results of this study. This prediction needs to be validated or modified with numerous new experimental results. Thus further research in the field is strongly recommended since there is a few number of available studies. Despite the effects of the presence of short fibers on the free shrinkage and long term loading of FRC beams remain an important engineering challenge for researchers.

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